Section-A

1. A metal sheet is inserted between the plates of a parallel plate capacitor of capacitance C. If the sheet partly occupies the space between the plates, the capacitance:

(A) remains C

(B) becomes greater than C

(C) becomes less than C

(D) becomes zero

Correct Answer: (B) becomes greater than C —(1)

Solution: When a metal sheet is inserted between the plates of a parallel plate capacitor, it effectively reduces the distance between the plates. The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 A}{d}$$

Where: - C is the capacitance, - ε_0 is the permittivity of free space, - A is the area of the plates, - d is the distance between the plates.

By inserting a metal sheet between the plates, the effective distance d_{eff} between the plates is reduced. As a result, the capacitance increases because the capacitance is inversely proportional to the distance between the plates.

The presence of the metal sheet increases the effective area for charge storage, and it also allows the plates to store more charge for the same applied voltage, leading to an increase in the overall capacitance.

Thus, the capacitance becomes greater than C.

Quick Tip

Inserting a dielectric material, such as a metal sheet, between the plates of a parallel plate capacitor decreases the effective distance between the plates and increases the capacitance. This happens because the metal sheet alters the electric field between the plates, leading to more charge storage capability.

2. **The electric field at a point in a region is given by $\vec{E} = \alpha\frac{\hat{r}}{r^3}$, where α is a constant and r is the distance of the point from the origin. The magnitude of potential of the point is:**

(A) $\frac{\alpha}{r}$

(B) $\frac{\alpha r^2}{2}$

(C) $\frac{\alpha}{2r^2}$

(D) $-\frac{\alpha}{r}$

Correct Answer: (A) $\frac{\alpha}{r}$ —(1)

Solution: We know that the electric field is the negative gradient of the potential:

$$\vec{E} = -\nabla V$$

The given electric field is:

$$\vec{E} = \alpha\frac{\hat{r}}{r^3}$$

Since the electric field is radial and only depends on r, we consider the radial component of the electric field:

$$E_r = \alpha\frac{1}{r^3}$$

The relationship between the electric field and the potential in one dimension is:

$$E_r = -\frac{dV}{dr}$$

Substituting the given electric field:

$$\alpha\frac{1}{r^3} = -\frac{dV}{dr}$$

Now, we integrate both sides with respect to r:

$$dV = -\alpha\frac{dr}{r^3}$$

Integrating both sides:

$$V(r) = \int \alpha\frac{dr}{r^3} = \frac{\alpha}{2r^2} + C$$

Where C is the constant of integration. In the context of electrostatics, we usually set the potential to zero at infinity, implying $C = 0$. Therefore:

$$V(r) = \frac{\alpha}{r}$$

Thus, the magnitude of the potential is $\frac{\alpha}{r}$, which is option (A).

Quick Tip

When the electric field is given in terms of r, integrate it with respect to r to find the potential. The negative sign indicates that the potential decreases in the direction of the electric field.

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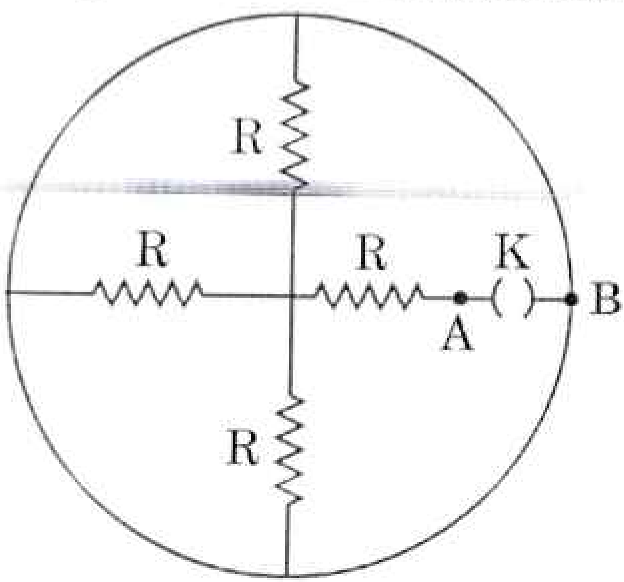
3. Four resistors, each of resistance R and a key K are connected as shown in the figure. The equivalent resistance between points A and B when key K is open will be:



(A) $4R$

(B) ∞

(C) $\frac{R}{4}$

(D) $\frac{3R}{2}$

Correct Answer: (D) $\frac{3R}{2}$ — (1)

Solution:

When key K is open, we observe that the resistors form a combination of series and parallel resistances. The two resistors connected between points A and B are in parallel, and the

Other two resistors are in series with this parallel combination.

1. The two resistors between A and B are in parallel, so their equivalent resistance is given by:

$$R_{eq} = \frac{R}{2}$$

2. This equivalent resistance is in series with the other resistor R, so the total equivalent resistance becomes:

$$R_{total} = R + \frac{R}{2} = \frac{3R}{2}$$

Thus, the equivalent resistance between points A and B when key K is open is $\frac{3R}{2}$.

Quick Tip

When dealing with resistors in series and parallel, simplify the circuit step by step. First, combine resistors in series or parallel, then combine the resulting resistances. This method makes the problem easier to solve.

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4. A charged particle gains a speed of 10^6 ms^{-1} when accelerated from rest through a potential difference of 10 kV. It enters a region of magnetic field of 0.4 T such that $\vec{v} \perp \vec{B}$. The radius of the circular path described by it is:

(A) 2.5 cm

(B) 5 cm

(C) 8 cm

(D) 10 cm

—(1)

Correct Answer: (B) 5 cm

Solution: When a charged particle moves in a magnetic field at a perpendicular angle to the field, it follows a circular path. The radius r of this circular path is given by the formula:

$$r = \frac{mv}{qB}$$

Where: - m is the mass of the particle, - v is the speed of the particle, - q is the charge of the particle, - B is the magnetic field strength.

We know the following: - The particle gains a speed $v = 10^6 \, \text{ms}^{-1}$ after being accelerated through a potential difference $V = 10 \, \text{kV} = 10^4 \, \text{V}$. - The magnetic field $B = 0.4 \, \text{T}$. - The energy gained by the particle is equal to the work done by the electric field, which can be expressed as:

$$\frac{1}{2}mv^2 = qV$$

From this equation, we can solve for m (mass of the particle) in terms of q (charge of the particle) and v:

$$m = \frac{2qV}{v^2}$$

Substituting this expression for m into the formula for r:

$$r = \frac{\left(\frac{2qV}{v^2}\right)v}{qB} = \frac{2V}{vB}$$

Now, substitute the given values: - $V = 10^4\,\mathrm{V}$, - $v = 10^6\,\mathrm{ms^{-1}}$, - $B = 0.4\,\mathrm{T}$:

$$r = \frac{2 \times 10^4}{10^6 \times 0.4} = \frac{2 \times 10^4}{4 \times 10^5} = 0.05\,\mathrm{m} = 5\,\mathrm{cm}$$

Thus, the radius of the circular path described by the particle is 5 cm. Therefore, the correct answer is option (B).

Quick Tip

For a charged particle moving in a magnetic field perpendicular to the field, the radius of the circular path can be found using the formula $r = \frac{mv}{qB}$, where v is the velocity, q is the charge, and B is the magnetic field strength.

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5. A current of $\frac{10}{\pi}$ A is maintained in a circular loop of radius 14 cm. The value of dipole moment associated with the loop is:

(A) $0.019\,\mathrm{Am^2}$

(B) $0.14\,\mathrm{Am^2}$

(C) $0.196\,\mathrm{Am^2}$

(D) $0.615\,\mathrm{Am^2}$

Correct Answer: (C) $0.196\,\mathrm{Am^2}$ — (1)

Solution: The magnetic dipole moment μ of a current-carrying loop is given by the formula:

$$\mu = I \cdot A$$

Where: - I is the current flowing through the loop, - A is the area of the loop.

For a circular loop, the area A is given by:

$$A = \pi r^2$$

Where r is the radius of the loop.

Given: - $I = \frac{10}{\pi}$ A, - $r = 14\,\text{cm} = 0.14\,\text{m}$.

First, calculate the area A:

$$A = \pi(0.14)^2 = \pi \times 0.0196 = 0.0616\,\text{m}^2$$

Now, calculate the magnetic dipole moment:

$$\mu = I \cdot A = \frac{10}{\pi} \times 0.0616 = 0.196\,\text{Am}^2$$

Thus, the dipole moment associated with the loop is $0.196\,\text{Am}^2$, which is option (C).

Quick Tip

To find the magnetic dipole moment of a current loop, use the formula $\mu = I \cdot A$, where A is the area of the loop. For a circular loop, $A = \pi r^2$.

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6. The magnetic flux linked with a coil changes with time t as $\phi = (8t^2 + 5t + 7)$, where t is in seconds and ϕ is in Wb. The value of emf induced in the coil at $t = 4$ s is:

(A) 32 V

(B) 37 V

(C) 64 V

(D) 69 V

— (1)

Correct Answer: (D) 69 V

Solution: The induced emf in a coil is given by Faraday's law of electromagnetic induction, which states that:

$$\text{emf} = -\frac{d\phi}{dt}$$

Where: - ϕ is the magnetic flux, - $\frac{d\phi}{dt}$ is the rate of change of flux.

The given magnetic flux is:

$$\phi = 8t^2 + 5t + 7$$

To find the induced emf, we need to differentiate the flux with respect to time t:

$$\frac{d\phi}{dt} = \frac{d}{dt}(8t^2 + 5t + 7)$$

Differentiating each term:

$$\frac{d\phi}{dt} = 16t + 5$$

Now, substitute $t = 4$ seconds into this expression to find the induced emf at that time:

$$\frac{d\phi}{dt} = 16(4) + 5 = 64 + 5 = 69\text{ V}$$

Thus, the induced emf in the coil at $t = 4$ s is 69 V. Therefore, the correct answer is option (D).

Quick Tip

To find the induced emf, differentiate the magnetic flux with respect to time. The induced emf is equal to the rate of change of magnetic flux.

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7. Which of the following rays coming from the Sun plays an important role in maintaining the Earth's warmth?

(A) Infrared rays

(B) γ rays

(C) UV rays

(D) Visible light rays

Correct Answer: (A) Infrared rays

— (1)

Solution: The Sun emits a wide range of electromagnetic radiation, including visible light, ultraviolet (UV) rays, infrared rays, and others. However, infrared rays are the primary radiation that contributes to the Earth's warmth.

- Infrared rays are absorbed by the Earth and re-radiated as heat, which is responsible for maintaining the Earth's temperature. This is often referred to as the greenhouse effect.
- While visible light contributes to illumination, and UV rays have other effects such as causing sunburns, it is the infrared radiation that plays a key role in heating the Earth's surface and atmosphere.

Thus, the correct answer is option (A), Infrared rays.

Quick Tip

Infrared rays, which are emitted by the Sun, are responsible for heating the Earth's surface. These rays have longer wavelengths than visible light and are essential in maintaining the Earth's warmth through the greenhouse effect.

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8. The dimensions of $(\mu\epsilon)^{-1}$, where ϵ is permittivity and μ is permeability of a medium, are:

(A) $[M^0L^1T^{-1}]$

(B) $[M^0L^2T^{-2}]$

(C) $[M^1L^2T^{-2}]$

(D) $[M^1L^{-1}T^1]$

Correct Answer: (B) $[M^0L^2T^{-2}]$ —(1)

To find the dimensions of $(\mu\epsilon)^{-1}$, we first need to understand the dimensions of permittivity (ϵ) and permeability (μ) in a medium. The permittivity of free space is given by the formula:

$$\epsilon = \frac{1}{\mu_0 c^2}$$

where μ_0 is the permeability of free space and c is the speed of light. The dimensions of ϵ are given as:

$$[\epsilon] = [M^{-1}L^{-3}T^{4}A^2]$$

The permeability of free space is given by:

$$\mu = \frac{1}{\epsilon_0 c^2}$$

and its dimensions are:

$$[\mu] = [MLT^{-2}A^{-2}]$$

Now, we can find the dimensions of the product $\mu\epsilon$:

$$[\mu\epsilon] = [MLT^{-2}A^{-2}] \times [M^{-1}L^{-3}T^4A^2] = [M^0L^{-2}T^2A^0]$$

Therefore, the dimensions of $(\mu\epsilon)^{-1}$ are the inverse of these dimensions:

$$(\mu\epsilon)^{-1} = [M^0L^2T^{-2}A^0] = [L^2T^{-2}]$$

Thus, the dimensions of $(\mu\epsilon)^{-1}$ are:

$$\boxed{[L^2T^{-2}]}$$

Step by Step Solution:

Step 1: Identify the formulas for permittivity and permeability:

$$\epsilon = \frac{1}{\mu_0 c^2}, \quad \mu = \frac{1}{\epsilon_0 c^2}$$

Step 2: Determine the dimensions of permittivity:

$$[\epsilon] = [M^{-1}L^{-3}T^4A^2]$$

Step 3: Determine the dimensions of permeability:

$$[\mu] = [MLT^{-2}A^{-2}]$$

Step 4: Calculate the dimensions of the product $\mu\epsilon$:

$$[\mu\epsilon] = [MLT^{-2}A^{-2}] \times [M^{-1}L^{-3}T^4A^2] = [M^0L^{-2}T^2A^0]$$

Step 5: Find the dimensions of $(\mu\epsilon)^{-1}$:

$$(\mu\epsilon)^{-1} = [M^0L^2T^{-2}A^0] = [M^0L^2T^{-2}]$$

Quick Tip

To calculate the dimensions of a product or quotient of physical quantities, multiply or divide their respective dimensions. For inverse quantities, simply invert the dimensions.

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9. Which of the following electromagnetic waves has photons of largest momentum?

(A) X-rays

(B) AM radio waves

(C) Microwaves

(D) TV waves

Correct Answer: (A) X-rays — (1)

Solution: The momentum p of a photon is related to its energy E by the formula:

$$p = \frac{E}{c}$$

Where: - p is the photon's momentum, - E is the energy of the photon, - c is the speed of light.

The energy of a photon is also related to its frequency ν by:

$$E = h\nu$$

Where h is Planck's constant. Combining the two equations, we get the momentum of a photon as:

$$p = \frac{h\nu}{c}$$

Since the frequency ν is directly proportional to the photon's momentum, the wave with the highest frequency will have the highest momentum.

- X-rays have the highest frequency among the options listed (compared to AM radio waves, microwaves, and TV waves). - Therefore, X-rays have the photons with the largest momentum.

Thus, the correct answer is option (A), X-rays.

Quick Tip

The momentum of a photon is directly proportional to its frequency. Since X-rays have the highest frequency, their photons carry the largest momentum.

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10. A compound microscope has an objective and an eyepiece of focal lengths f_0 and f_e, respectively. To obtain a large magnification of a small object, the microscope should have:

(A) f_0 and f_e small, and $f_e > f_0$

(B) f_0 and f_e small, and $f_0 > f_e$

(C) f_0 and f_e large, and $f_e > f_0$

(D) f_0 and f_e large, and $f_0 > f_e$

Correct Answer: (A) f_0 and f_e small, and $f_e > f_0$ — (1)

Solution: The magnification M of a compound microscope is given by the formula:

$$M = \frac{\text{angular magnification of the eyepiece} \times \text{magnification of the objective}}{1}$$

The magnification of the objective is given by:

$$M_o = \frac{D}{f_0}$$

Where: - D is the least distance of distinct vision (usually taken as 25 cm), - f_0 is the focal length of the objective lens.

The magnification of the eyepiece is given by:

$$M_e = \frac{D}{f_e}$$

Where f_e is the focal length of the eyepiece.

To achieve large magnification, both f_0 and f_e should be small. The objective lens's focal length f_0 should be small to produce a larger magnification, and the eyepiece's focal length f_e should be slightly larger than f_0, since the eyepiece is used to magnify the image formed by the objective.

Thus, to obtain a large magnification, the microscope should have both f_0 and f_e small, and $f_e > f_0$. Therefore, the correct answer is option (A).

Quick Tip

For a compound microscope, smaller focal lengths of the objective and eyepiece lead to greater magnification. A larger focal length for the eyepiece compared to the objective ensures an efficient formation of a magnified image.

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11. Two coherent light waves, each having amplitude 'a', superpose to produce an interference pattern on a screen. The intensity of light as seen on the screen varies between:

(A) 0 and $2a^2$

(B) 0 and $4a^2$

(C) a^2 and $2a^2$

(D) $2a^2$ and $4a^2$

Correct Answer: (B) 0 and $4a^2$ — (1)

Solution: In interference, the intensity of light varies depending on the phase difference between the two coherent waves. The intensity of the resulting wave is given by the formula:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi)$$

Where: - I_1 and I_2 are the intensities of the individual waves, - ϕ is the phase difference between the waves.

For waves with equal amplitude a, the intensity of each wave is proportional to the square of the amplitude:

$$I_1 = I_2 = a^2$$

Thus, the total intensity becomes:

$$I = 2a^2 + 2a^2\cos(\phi)$$

Now, the intensity varies with the phase difference ϕ. The maximum intensity occurs when

$\cos(\phi) = 1$ (constructive interference), and the minimum intensity occurs when $\cos(\phi) = -1$ (destructive interference).

- For maximum intensity (constructive interference):

$$I_{\text{max}} = 2a^2 + 2a^2 = 4a^2$$

- For minimum intensity (destructive interference):

$$I_{\text{min}} = 2a^2 - 2a^2 = 0$$

Thus, the intensity of light varies between 0 and $4a^2$, which corresponds to option (B).

Quick Tip

The intensity in an interference pattern depends on the phase difference between the waves. Constructive interference leads to maximum intensity, while destructive interference leads to minimum intensity.

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12. The kinetic energy of an alpha particle is four times the kinetic energy of a proton. The ratio $\left(\frac{\lambda_\alpha}{\lambda_p}\right)$ of de Broglie wavelengths associated with them will be:

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{1}{4}$ — (D)

Solution: The de Broglie wavelength λ of a particle is related to its momentum p by the equation:

$$\lambda = \frac{h}{p}$$

Where: - h is Planck's constant, - p is the momentum of the particle.

Momentum is related to the kinetic energy K and mass m by the equation:

$$K = \frac{p^2}{2m}$$

Thus, the momentum can be expressed as:

$$p = \sqrt{2mK}$$

The de Broglie wavelength then becomes:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Now, we are told that the kinetic energy of the alpha particle is four times that of the proton:

$$K_\alpha = 4K_p$$

The ratio of the de Broglie wavelengths is:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\frac{h}{\sqrt{2m_\alpha K_\alpha}}}{\frac{h}{\sqrt{2m_p K_p}}}$$

Simplifying this expression:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{2m_p K_p}}{\sqrt{2m_\alpha K_\alpha}} = \frac{\sqrt{m_p K_p}}{\sqrt{m_\alpha K_\alpha}}$$

Since the mass of the alpha particle is four times that of the proton $m_\alpha = 4m_p$, and $K_\alpha = 4K_p$, we get:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{m_p \cdot K_p}}{\sqrt{4m_p \cdot 4K_p}} = \frac{\sqrt{m_p K_p}}{\sqrt{16 m_p K_p}} = \frac{1}{4}$$

Thus, the ratio of the de Broglie wavelengths is $\frac{1}{4}$. Therefore, the correct answer is option (C).

Quick Tip

The de Broglie wavelength is inversely proportional to the momentum of the particle. Since the kinetic energy is proportional to the square of the momentum, the de Broglie wavelength will be smaller for particles with higher kinetic energy.

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13. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C), and (D) as given below.

Assertion (A): The impurities in p-type Si are not pentavalent atoms.

Reason (R): The hole density in the valence band in a p-type semiconductor is almost equal to the acceptor density.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

—(1)

Solution: - Assertion (A): In p-type semiconductors, the majority charge carriers are holes, which are created by doping with trivalent elements (such as boron), not pentavalent elements. Pentavalent elements (like phosphorus) are used in n-type semiconductors to provide extra electrons. Therefore, the assertion that impurities in p-type Si are not pentavalent atoms is true.

- Reason (R): In p-type semiconductors, the acceptor atoms (such as boron) create holes in the valence band. The number of holes generated is approximately equal to the number of acceptor atoms, leading to the conclusion that the hole density is almost equal to the acceptor density. Hence, Reason (R) is also true.

However, Reason (R) does not explain Assertion (A) directly because the hole density in p-type semiconductors being equal to the acceptor density is not related to whether the dopant is pentavalent or trivalent. The assertion is about the type of dopant (trivalent for p-type), while the reason is a property of the hole density in p-type materials.

Thus, the correct answer is option (B): Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

Quick Tip

In semiconductors, p-type doping involves adding trivalent elements (like boron), while n-type doping involves pentavalent elements (like phosphorus). The hole density in p-type semiconductors is nearly equal to the acceptor density.

14. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

Assertion (A): During the formation of a nucleus, the mass defect produced is the source of the binding energy of the nucleus.

Reason (R): For all nuclei, the value of binding energy per nucleon increases with mass number.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false. — (1)

Solution: - Assertion (A): The mass defect in the formation of a nucleus is indeed the source of the binding energy. The mass defect refers to the difference between the mass of the nucleus and the sum of the masses of the individual nucleons. This missing mass is converted into binding energy according to Einstein's equation $E = \Delta mc^2$. Therefore, Assertion (A) is true.

- Reason (R): The binding energy per nucleon does not continuously increase with mass number for all nuclei. While it does increase with mass number for light nuclei, it reaches a maximum value for medium-sized nuclei (around iron-56) and then decreases for heavier nuclei. Thus, Reason (R) is false because the binding energy per nucleon does not increase for all nuclei.

Therefore, the correct answer is option (C): Assertion (A) is true, but Reason (R) is false.

Quick Tip

The binding energy per nucleon increases with mass number for light nuclei, but it reaches a peak for medium-sized nuclei like iron. For heavier nuclei, the binding energy per nucleon decreases.

15. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C), and (D) as given below.

Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

Assertion (A): The Balmer series in the hydrogen atom spectrum is formed when the electron jumps from a higher energy state to the ground state.

Reason (R): In Bohr's model of the hydrogen atom, the electron can jump between successive orbits only.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false. —(1)

Solution: - Assertion (A): The Balmer series in the hydrogen atom spectrum is formed when the electron transitions from a higher energy state to the $n = 2$ energy level, not necessarily the ground state. This is because the Balmer series corresponds to transitions where the final state is $n = 2$, not $n = 1$. Therefore, Assertion (A) is false.

- Reason (R): In Bohr's model of the hydrogen atom, the electron can indeed jump between any two allowed orbits, not just successive orbits. The electron can jump between any two levels, provided that the energy difference matches the energy of the emitted or absorbed photon. Therefore, Reason (R) is also false.

Thus, both Assertion (A) and Reason (R) are false, and the correct answer is option (D).

Quick Tip

In the Bohr model of the hydrogen atom, the electron can jump between any two orbits, and the Balmer series corresponds to transitions where the final state is $n = 2$, not the ground state $n = 1$.

16. **Assertion (A):** In Rutherford's alpha particle scattering experiment, the presence of only few alpha particles at angle of scattering π led him to the discovery of the nucleus.

Reason (R): The size of the nucleus is approximately 10^{-5} times the size of an atom and therefore only few alpha particles are rebounded.

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). — (1)

Solution: - Assertion (A): In Rutherford's alpha particle scattering experiment, he observed that most of the alpha particles passed through the gold foil, but a few were deflected at very large angles, some even rebounding. This led him to conclude that most of the atom is empty space, with a small, dense nucleus at its center. This observation was crucial in the discovery of the atomic nucleus. Hence, Assertion (A) is true.

- Reason (R): The reason for only a few alpha particles being deflected at large angles is that the nucleus is extremely small compared to the overall size of the atom. The nucleus is about 10^{-5} times the size of the atom, so most of the alpha particles pass through the empty space around the nucleus, and only a few collide with the dense nucleus, leading to large angle scattering or rebound. Therefore, Reason (R) is also true and correctly explains Assertion (A).

Thus, the correct answer is option (A): Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

Quick Tip

In Rutherford's alpha particle scattering experiment, the deflection of alpha particles at large angles provided evidence of the existence of a dense nucleus. The small size of the nucleus explains why only a few alpha particles were deflected significantly.

Section-B

17. The threshold frequency for a given metal is 3.6×10^{14} Hz. If monochromatic radiations of frequency 6.8×10^{14} Hz are incident on this metal, find the cut-off potential for the photoelectrons.

- Threshold frequency, $\nu_0 = 3.6 \times 10^{14}$ Hz
- Frequency of incident radiation, $\nu = 6.8 \times 10^{14}$ Hz
- Planck's constant, $h = 6.63 \times 10^{-34}\,\text{J} \cdot \text{s}$
- Charge of the electron, $e = 1.6 \times 10^{-19}\,\text{C}$

Solution: According to Einstein's photoelectric equation:

$$K_{\text{max}} = h(\nu - \nu_0)$$

Where K_{max} is the maximum kinetic energy of the photoelectrons.
The kinetic energy of the photoelectrons is related to the cut-off potential $V_{\text{cut-off}}$ by:

$$K_{\text{max}} = eV_{\text{cut-off}}$$

Therefore, we can write:

$$eV_{\text{cut-off}} = h(\nu - \nu_0)$$

1/2

Substituting the given values:

$$V_{\text{cut-off}} = \frac{h(\nu - \nu_0)}{e}$$

- 1/2

Substitute $h = 6.63 \times 10^{-34}\,\text{J} \cdot \text{s}$, $e = 1.6 \times 10^{-19}\,\text{C}$, $\nu = 6.8 \times 10^{14}$ Hz, and $\nu_0 = 3.6 \times 10^{14}$ Hz:

$$V_{\text{cut-off}} = \frac{(6.63 \times 10^{-34}) \times (6.8 \times 10^{14} - 3.6 \times 10^{14})}{1.6 \times 10^{-19}}$$ — 1/2

$$V_{\text{cut-off}} = \frac{6.63 \times 10^{-34} \times 3.2 \times 10^{14}}{1.6 \times 10^{-19}}$$

$$V_{\text{cut-off}} = \frac{2.12 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.33\,\text{V}$$ — 1/2

Thus, the cut-off potential for the photoelectrons is 1.33 V.

Quick Tip

The cut-off potential is obtained from the difference between the frequency of the incident light and the threshold frequency. The greater the frequency difference, the higher the energy of the emitted photoelectrons.

---

18. (a) A point object is placed in air at a distance $\frac{R}{3}$ in front of a convex surface of radius of curvature R, separating air from a medium of refractive index n (where $n < 4$). Find the nature and position of the image formed.

Solution:

The focal length f of a spherical surface separating two media is given by the formula:

$$\frac{n_2 - n_1}{f} = \frac{n_2}{R}$$

where: - $n_2 = n$ (refractive index of the medium), - $n_1 = 1$ (refractive index of air), - R is the radius of curvature of the convex surface.

Substituting the values:

$$\frac{n-1}{f} = \frac{n}{R}$$

$$f = \frac{R}{n-1}$$ 1/2

Now, using the mirror equation:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

1/2

Where: - $u = -\frac{R}{3}$ (object distance), - v is the image distance (which we need to find).

Substituting $u = -\frac{R}{3}$ and $f = \frac{R}{n-1}$:

1/2

$$\frac{1}{\frac{R}{n-1}} = \frac{1}{v} + \frac{3}{R}$$

Solving for v:

$$v = \frac{R(n-1)}{3n-4}$$

1/2

Thus, the position of the image is given by:

$$v = \frac{R(n-1)}{3n-4}$$

This gives the image position based on the refractive index n and the radius of curvature R.

Quick Tip

In problems involving spherical surfaces, always apply the mirror equation carefully, and remember to use the sign convention for distances. The focal length for a spherical surface separating two media can be derived using the lens maker's formula.

(b) In Young's double slit experimental set-up, the intensity of the central maximum is I_0. Calculate the intensity at a point where the path difference between two interfering waves is $\frac{\lambda}{3}$.

Solution:

The intensity in Young's double slit experiment is given by:

$$I = I_0 \cos^2\left(\frac{\pi \Delta x}{\lambda}\right)$$

1/2

Where: - I_0 is the intensity of the central maximum, - Δx is the path difference between the two waves, - λ is the wavelength of the light.

Given that the path difference $\Delta x = \frac{\lambda}{3}$, we substitute this into the equation:

$$I = I_0 \cos^2\left(\frac{\pi \times \frac{\lambda}{3}}{\lambda}\right)$$ $-1/2$

Simplifying:

$$I = I_0 \cos^2\left(\frac{\pi}{3}\right)$$ $1/2$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, we get:

$$I = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Thus, the intensity at the point where the path difference is $\frac{\lambda}{3}$ is:

$$I = \frac{I_0}{4}$$ $1/2$

Quick Tip

In Young's double slit experiment, the intensity at any point depends on the path difference between the two waves. If the path difference is a fraction of the wavelength, the intensity can be calculated using the formula $I = I_0 \cos^2\left(\frac{\pi \Delta x}{\lambda}\right)$.

---

19. A voltmeter of resistance 1000 Ω can measure up to 25 V. How will you convert it so that it can read up to 250 V?

Solution:

To convert the voltmeter so that it can measure up to 250 V, we use a series resistor.

Let the series resistance be R_s, and the total resistance should be such that the voltmeter can measure 250 V when the original meter reads 25 V.

Using the formula for voltage division:

$$V_{\text{new}} = V_{\max} \times \frac{R_{\text{meter}}}{R_{\text{meter}} + R_s}$$ $1/2$

Where: - V_{new} is the new voltage range (250 V), - $V_{\max}$ is the original maximum voltage (25 V), - R_{meter} is the resistance of the voltmeter (1000 Ω).

We can rearrange the formula to solve for R_s:

$$\frac{250}{25} = \frac{1000}{1000 + R_s}$$

1/2

Simplifying:

$$10 = \frac{1000}{1000 + R_s}$$

Now, solve for R_s:

$$10(1000 + R_s) = 1000$$

1/2

$$10000 + 10R_s = 1000$$

$$10R_s = 1000 - 10000 = 9000$$

$$R_s = 9000\,\Omega$$

1/2

Thus, the required series resistance is $9000\,\Omega$.

Quick Tip

To extend the voltage range of a voltmeter, a series resistor is used. The value of the series resistor is calculated by the voltage division rule. Make sure to use a resistor with appropriate power rating to avoid overheating.

---

20. When a neutron collides with $^{235}_{92}$U, the nucleus gives $^{140}_{54}$Xe and $^{94}_{38}$Sr as fission products, and two neutrons are ejected. Calculate the mass defect and the energy released (in MeV) in the process. Given:

$m(^{235}_{92}\text{U}) = 235.04393\,\text{u}$, $m(^{140}_{54}\text{Xe}) = 139.92164\,\text{u}$, $m(^{94}_{38}\text{Sr}) = 93.91536\,\text{u}$, $m(^{1}_{0}n) = 1.00866\,\text{u}$.

$1\ \text{u} = 931\ \text{MeV/c}^2$

Solution: In this fission process, the total mass before and after the reaction changes. The total mass defect Δm is the difference between the mass of the fission products and the initial mass.

$$\Delta m = m(\text{Initial mass}) - m(\text{Final mass})$$

The initial mass is the mass of the $^{235}_{92}$U nucleus plus the mass of the neutron:

$$m_{\text{initial}} = m(^{235}_{92}\text{U}) + m(^{1}_{0}n)$$

Substituting the given values:

$$m_{\text{initial}} = 235.04393 + 1.00866 = 236.05259\,\text{u}$$ 1/2

The final mass is the mass of the fission products (the $^{140}_{54}$Xe and $^{94}_{38}$Sr nuclei) plus the mass of the two neutrons:

$$m_{\text{final}} = m(^{140}_{54}\text{Xe}) + m(^{94}_{38}\text{Sr}) + 2 \times m(^{1}_{0}n)$$ 1/2

Substituting the given values:

$$m_{\text{final}} = 139.92164 + 93.91536 + 2 \times 1.00866 = 235.85432\,\text{u}$$

Now, the mass defect is:

$$\Delta m = 236.05259 - 235.85432 = 0.19827\,\text{u}$$ 1/2

To find the energy released, we use the equivalence $E = \Delta m \times 931\,\text{MeV}/c^2$:

$$E = 0.19827 \times 931 = 184.59\,\text{MeV}$$ 1/2

Thus, the energy released in the process is $\boxed{184.59}$ MeV.

Quick Tip

The mass defect in a nuclear reaction is the difference in mass between the initial nucleus and the sum of the masses of the products. This mass defect is converted into energy, which can be calculated using the equation $E = \Delta m \times 931\ \text{MeV}/c^2$.

---

21. The resistance of a wire at 25°C is 10.0 Ω. When heated to 125°C, its resistance becomes 10.5 Ω. Find (i) the temperature coefficient of resistance of the wire, and (ii) the resistance of the wire at 425°C.

Solution:

The temperature dependence of the resistance of a conductor is given by the formula:

$$R_t = R_0[1 + \alpha(t - t_0)]$$ — 1/2

Where: - R_t is the resistance at temperature t, - R_0 is the resistance at a reference temperature t_0, - α is the temperature coefficient of resistance, - t is the temperature at which the resistance is measured, - t_0 is the reference temperature.

Given: - $R_0 = 10.0\,\Omega$ (resistance at 25°C), - $R_{125} = 10.5\,\Omega$ (resistance at 125°C), - $t_0 = 25C$, - $t = 125C$.

(i) To find the temperature coefficient of resistance α:

Using the formula:

$$R_{125} = R_0[1 + \alpha(125 - 25)]$$ — 1/2

Substitute the known values:

$$10.5 = 10.0[1 + \alpha(100)]$$

Simplifying:

$$1.05 = 1 + 100\alpha$$

Solving for α:

$$100\alpha = 0.05$$

$$\alpha = \frac{0.05}{100} = 0.0005\,\text{per}^\circ\text{C}$$

Thus, the temperature coefficient of resistance is $\alpha = 0.0005\,\text{per}^\circ\text{C}$.

(ii) To find the resistance of the wire at 425°C, use the formula again:

$$R_{425} = R_0[1 + \alpha(425 - 25)]$$

Substitute the known values:

$$R_{425} = 10.0[1 + 0.0005(425 - 25)]$$ 1/2

$$R_{425} = 10.0[1 + 0.0005 \times 400]$$

$$R_{425} = 10.0[1 + 0.2]$$

1/2

$$R_{425} = 10.0 \times 1.2 = 12.0\,\Omega$$

Thus, the resistance of the wire at 425°C is $12.0\,\Omega$.

Quick Tip

To calculate the resistance at a different temperature, use the formula $R_t = R_0[1 + \alpha(t - t_0)]$, where α is the temperature coefficient of resistance. This formula assumes the material's temperature coefficient is constant over the given temperature range.

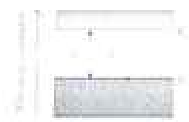
Section-B

22. (a) Draw the energy-band diagram for conductors, semiconductors, and insulators at T = 0 K. How is an electron-hole pair formed in a semiconductor at room temperature?

Solution:

CONDUCTORS

1/4



SEMICONDUCTORS

1/4

INSULATORS

1/4

At $T = 0$ K, the energy bands for conductors, semiconductors, and insulators can be described as follows:

- Conductors: In conductors, the conduction band overlaps with the valence band, meaning electrons can freely move in the material, allowing electrical conduction even at absolute zero temperature. 1/4
- Semiconductors: In semiconductors, the valence band is completely filled, and there is a small energy gap (band gap) between the valence band and the conduction band. At $T = 0$ K, no electrons are in the conduction band. 1/4
- Insulators: In insulators, the band gap between the valence band and conduction band is large, and electrons cannot move to the conduction band at $T = 0$ K, preventing conduction.

At room temperature (above 0 K), thermal energy excites some electrons from the valence band to the conduction band, leaving behind holes in the valence band. This creates electron-hole pairs. These electron-hole pairs are responsible for electrical conduction in semiconductors. 1/4

Energy Band Diagram at T = 0 K:

Conductors:	No band gap, conduction and valence bands overlap.
Semiconductors:	Small band gap, electrons at absolute zero fill the valence band, conduction band i
Insulators:	Large band gap, electrons are in the valence band.

Quick Tip

In semiconductors, at room temperature, the thermal energy excites some electrons from the valence band to the conduction band, leaving behind holes. This forms electron-hole pairs, which contribute to electrical conduction.

(b) Carbon and silicon both are members of the IV group of the periodic table and have the same lattice structure. Carbon is an insulator whereas silicon is a semiconductor. Explain.

Solution:

While both carbon and silicon are in the same group of the periodic table and have similar lattice structures, their electrical properties differ significantly.

- Carbon: In diamond (a form of carbon), the atoms are bonded in a three-dimensional lattice where each carbon atom forms four covalent bonds. This results in a large band gap between the valence band and the conduction band. The large band gap prevents electrons from easily moving to the conduction band, making diamond an insulator. 1½
- Silicon: Silicon also forms a similar covalent lattice, but the band gap between the valence band and conduction band is smaller compared to carbon. At room temperature, some electrons in silicon can gain enough energy to move into the conduction band, making it a semiconductor.

Thus, the main difference between carbon and silicon is the band gap: carbon has a large band gap (insulator), while silicon has a smaller band gap, allowing it to conduct electricity under certain conditions (semiconductor).

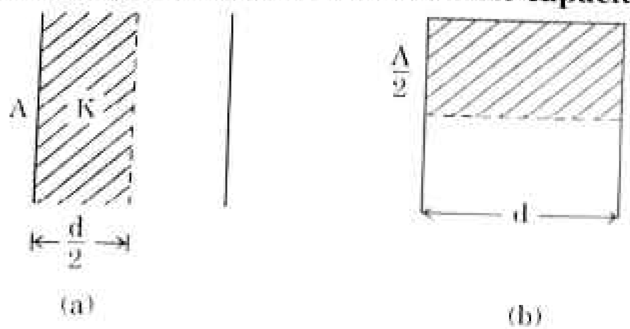
Quick Tip

The electrical properties of a material depend largely on the size of its band gap. Materials with large band gaps are insulators, while those with small band gaps are semiconductors.

---

23. A parallel plate capacitor has plate area A and plate separation d. Half of the space

between the plates is filled with a material of dielectric constant K in two ways as shown in the figure. Find the values of the capacitance of the capacitors in the two cases.



Solution:

For a parallel plate capacitor, the capacitance C is given by:

$$C = \frac{\epsilon_0 A}{d}$$

where: - ϵ_0 is the permittivity of free space, - A is the area of the plates, - d is the separation between the plates.

When a dielectric is inserted between the plates, the capacitance increases by a factor of K, the dielectric constant. The cases can be analyzed as follows:

Case (a) In this case, the capacitor is filled with the dielectric constant K in half of the space between the plates. We treat the capacitor as two capacitors in series, one filled with the dielectric and the other without it.

- The first capacitor has a dielectric material with constant K and a distance of $\frac{d}{2}$. - The second capacitor has no dielectric material and also has a distance of $\frac{d}{2}$.

The capacitance of the first capacitor is:

$$C_1 = \frac{K\epsilon_0 A}{d/2} = \frac{2K\epsilon_0 A}{d}$$ 1/2

The capacitance of the second capacitor is:

$$C_2 = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d}$$ 1/2

Since the capacitors are in series, the total capacitance is:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$ 1/2

Substituting the values of C_1 and C_2:

$$\frac{1}{C} = \frac{1}{\frac{2K\epsilon_0 A}{d}} + \frac{1}{\frac{2\epsilon_0 A}{d}}$$

Simplifying:

$$\frac{1}{C} = \frac{d}{2K\epsilon_0 A} + \frac{d}{2\epsilon_0 A}$$

$$\frac{1}{C} = \frac{d}{2\epsilon_0 A}\left(\frac{1}{K} + 1\right)$$

Thus, the total capacitance is:

$$C = \frac{2\epsilon_0 A}{d}\left(\frac{K}{K+1}\right)$$ 1/2

Case (b) In this case, the entire space between the plates is filled with the dielectric material with dielectric constant K. The capacitance is simply:

$$C = \frac{K\epsilon_0 A}{d}$$ 1

Quick Tip

When dielectric materials are inserted between the plates of a capacitor, the capacitance increases. For a parallel plate capacitor with different dielectric materials, we can use series and parallel combinations depending on the geometry of the dielectric placement.

---

24. In Young's double slit experiment, the separation between the two slits is 1.0 mm and the screen is 1.0 m away from the slits. A beam of light consisting of two wavelengths, 500 nm and 600 nm, is used to obtain interference fringes. Calculate:

(a) The distance between the first maxima for the two wavelengths.

Solution:

In Young's double slit experiment, the distance between the maxima is given by the formula:

$$y_m = \frac{m\lambda D}{d}$$ 1/2

Where: - y_m is the distance between the m^{th} maxima and the central maximum, - m is the order of the maxima (for first maxima, $m = 1$), - λ is the wavelength of the light, - D is the distance between the slits and the screen, - d is the separation between the slits.

For the two wavelengths:

- $\lambda_1 = 500\,\text{nm} = 500 \times 10^{-9}\,\text{m}$, - $\lambda_2 = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$, - $D = 1.0\,\text{m}$, - $d = 1.0\,\text{mm} = 1.0 \times 10^{-3}\,\text{m}$.

The distance between the first maxima for the two wavelengths:

For $\lambda_1 = 500\,\text{nm}$:

$$y_1 = \frac{1 \times 500 \times 10^{-9} \times 1.0}{1.0 \times 10^{-3}} = 5.0 \times 10^{-4}\,\text{m} = 0.5\,\text{mm}$$ 1/2

For $\lambda_2 = 600\,\text{nm}$:

$$y_2 = \frac{1 \times 600 \times 10^{-9} \times 1.0}{1.0 \times 10^{-3}} = 6.0 \times 10^{-4}\,\text{m} = 0.6\,\text{mm}$$

The distance between the first maxima for the two wavelengths is:

$$\Delta y = y_2 - y_1 = 0.6\,\text{mm} - 0.5\,\text{mm} = 0.1\,\text{mm}$$ 1/2

Thus, the distance between the first maxima for the two wavelengths is $0.1\,\text{mm}$.

(b) The least distance from the central maximum, where the bright fringes due to both the wavelengths coincide.

Solution:

For the bright fringes to coincide, the path difference for the two wavelengths must be an integer multiple of both wavelengths. This condition occurs when the path difference is equal to the least common multiple (LCM) of the wavelengths.

The condition for constructive interference is:

$$\Delta y = m\lambda_1 = n\lambda_2$$ 1/2

Where m and n are integers.

To find the least distance where both wavelengths give bright fringes at the same position, we need to find the least common multiple (LCM) of the two wavelengths.

The LCM of 500 nm and 600 nm is 3000 nm, which corresponds to the first coincidence.

Now, using the formula for the distance between fringes:

$$y = \frac{m\lambda D}{d}$$

Substituting for $\lambda = 3000\,\text{nm} = 3.0 \times 10^{-6}\,\text{m}$, $D = 1.0\,\text{m}$, and $d = 1.0 \times 10^{-3}\,\text{m}$:

$$y = \frac{1 \times 3.0 \times 10^{-6} \times 1.0}{1.0 \times 10^{-3}} = 3.0 \times 10^{-3}\,\text{m} = 3.0\,\text{mm} \qquad 1$$

Thus, the least distance from the central maximum where the bright fringes due to both wavelengths coincide is 3.0 mm.

Quick Tip

In Young's double slit experiment, the separation between maxima for different wavelengths is directly proportional to the wavelength. The least distance where the bright fringes coincide is obtained when the path difference is a multiple of both wavelengths.

---

25. Differentiate between half-wave and full-wave rectification. With the help of a circuit diagram, explain the working of a full-wave rectifier.

Solution:

Difference between Half-Wave and Full-Wave Rectification: (Any two points)

- **Number of Diodes:** Half-Wave Rectifier uses 1 diode, while Full-Wave Rectifier uses 2 diodes. ½

+

- **Operation:** In Half-Wave Rectification, conduction occurs during one half cycle of the AC input, whereas Full-Wave Rectification uses both positive and negative half cycles for conduction. ½

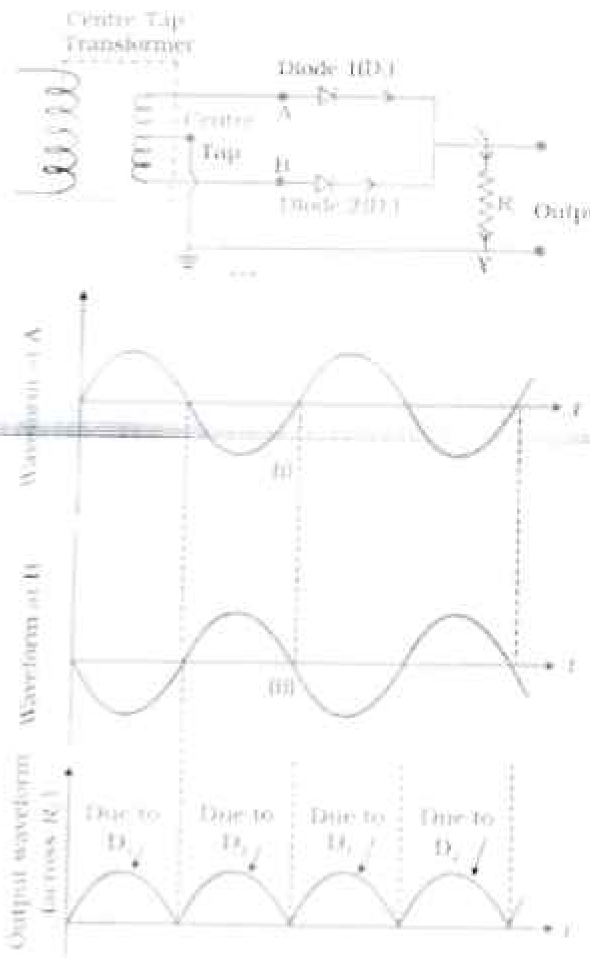
- **Efficiency:** Half-Wave Rectification has a lower efficiency of 40.6%, while Full-Wave Rectification has a higher efficiency of 81.2%.

- **Output:** Half-Wave Rectification produces pulsating DC with gaps, whereas Full-Wave Rectification produces smooth DC with no gaps.

- **Peak Inverse Voltage:** In Half-Wave Rectification, the peak inverse voltage is higher, whereas it is lower in Full-Wave Rectification.

Working of a Full-Wave Rectifier:

A full-wave rectifier consists of two diodes arranged in a bridge configuration. When an alternating current (AC) signal is applied to the input of the rectifier, both positive and negative cycles of the AC signal are used for rectification. During the positive half cycle, one diode conducts and the current flows through the load resistor in one direction, while during the negative half cycle, the other diode conducts and the current flows in the same direction through the load resistor.



1/2

1/2

Quick Tip

Full-wave rectifiers are more efficient than half-wave rectifiers as they use both halves of the input AC waveform.

26. An electron of mass m and charge $-e$ is revolving anticlockwise around the nucleus of an atom.

(a) Obtain the expression for the magnetic dipole moment μ of the atom.

Solution:

The magnetic dipole moment μ due to the revolving electron is given by:

$$\mu = I \cdot A$$

Where: - I is the current associated with the moving electron, - A is the area enclosed by the orbit of the electron.

Step 1: Current due to the electron The electron is moving in a circular orbit, so the current is defined as the charge passing through a point per unit time. The time period T of the electron's revolution is the time it takes to complete one full revolution around the nucleus. The frequency of the electron is:

$$f = \frac{v}{2\pi r}$$ 1/2

where: - v is the velocity of the electron, - r is the radius of the orbit.

The current I is given by:

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$$ 1/2

Step 2: Area enclosed by the orbit The area A enclosed by the electron's orbit is the area of a circle:

$$A = \pi r^2$$

Step 3: Magnetic dipole moment Now, the magnetic dipole moment is:

$$\mu = I \cdot A = \left(\frac{ev}{2\pi r}\right) \cdot (\pi r^2) = \frac{evr}{2}$$ 1/2

Thus, the expression for the magnetic dipole moment of the atom is:

$$\mu = \frac{evr}{2}$$ 1/2

(b) If $\vec{L}$ is the angular momentum of the electron, show that $\vec{\mu} = -\left(\frac{e}{2m}\right)\vec{L}$.

Solution:

The angular momentum of the electron $\vec{L}$ is given by:

$$\vec{L} = mvr$$ 1/2

Now, substitute vr from this equation into the expression for the magnetic dipole moment:

$$\mu = \frac{evr}{2} = \frac{e}{2m} \cdot (mvr) = -\frac{e}{2m} \cdot \vec{L}$$ 1/2

Thus, we have shown that:

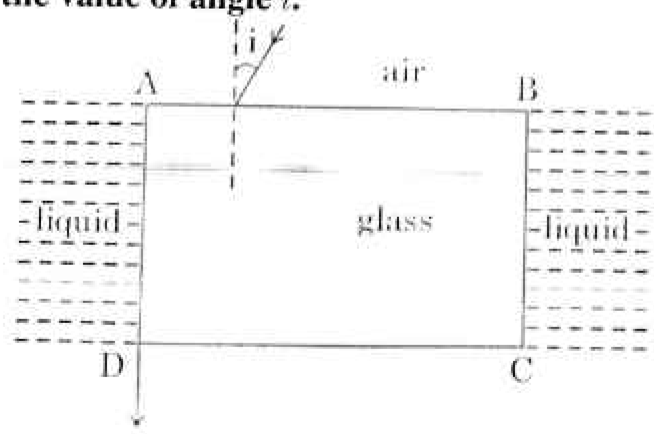
$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

Quick Tip

In the case of an electron moving in a circular orbit, the magnetic dipole moment is proportional to the angular momentum of the electron. The negative sign indicates that the direction of the magnetic dipole moment is opposite to the direction of the angular momentum due to the negative charge of the electron.

---

27. A rectangular glass slab ABCD (refractive index 1.5) is surrounded by a transparent liquid (refractive index 1.25) as shown in the figure. A ray of light is incident on face AB at an angle i such that it is refracted out grazing the face AD. Find the value of angle i.



Solution:

We use Snell's law to solve this problem, which relates the angle of incidence and refraction between two media:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$ 1/2

Where: - n_1 and n_2 are the refractive indices of the two media, - θ_1 is the angle of incidence in the first medium, - θ_2 is the angle of refraction in the second medium.

Step 1: Conditions for Grazing Refraction At the glass-liquid interface, the ray is refracted out grazing the face AD, meaning the angle of refraction at the glass-liquid interface must be 90°.

Step 2: Using Snell's Law at the Glass-Liquid Interface - The refractive index of glass is $n_{\text{glass}} = 1.5$, - The refractive index of the liquid is $n_{\text{liquid}} = 1.25$, - The angle of refraction $\theta_2 = 90^\circ$. 1/2

Using Snell's Law at the interface:

$$n_{\text{glass}} \sin i = n_{\text{liquid}} \sin 90^\circ$$ 1/2

$$1.5 \sin i = 1.25$$ 1/2

$$\sin i = \frac{1.25}{1.5}$$

$$\sin i = \frac{5}{6}$$ 1/2

Step 3: Calculating the Angle Now, calculate the angle of incidence i:

$$i = \sin^{-1}\left(\frac{5}{6}\right) \approx 56.44^\circ$$ 1/2

Thus, the angle of incidence $i \approx 56.44^\circ$.

Quick Tip

For grazing refraction, the angle of refraction is $90°$, and Snell's law can be used to determine the angle of incidence in the first medium.

---

28. (a) Two small solid metal balls A and B of radii R and $2R$ having charge densities 2 and 3 respectively are kept far apart. Find the charge densities on A and B after they are connected by a conducting wire.

Solution:

Let the charge densities on balls A and B before connecting by the wire be σ_A and σ_B, and after connecting the wire, let the charge densities be σ'_A and σ'_B.

The initial charge densities are given as: - On ball A: $\sigma_A = 2$ - On ball B: $\sigma_B = 3$

Step 1: Initial Charges on Balls The charge on a spherical ball is given by the formula:

$$Q = \sigma \cdot A = \sigma \cdot 4\pi r^2$$

Where: - r is the radius of the ball, - σ is the surface charge density, - $A = 4\pi r^2$ is the surface area of the ball.

Thus, the initial charges on A and B are:

For ball A:

$$Q_A = \sigma_A \cdot 4\pi R^2 = 2 \cdot 4\pi R^2 = 8\pi R^2$$

For ball B: 1/2

$$Q_B = \sigma_B \cdot 4\pi (2R)^2 = 3 \cdot 4\pi (4R^2) = 48\pi R^2$$

Step 2: Total Charge and Conservation of Charge When the balls are connected by a conducting wire, they will share charge until they reach the same potential. Since the potentials on the balls must be the same, we use the formula for the potential on a spherical ball:

$$V = \frac{kQ}{r}$$

Where: - k is Coulomb's constant, - Q is the charge on the ball, - r is the radius of the ball.

Let the charges on balls A and B after the connection be Q'_A and Q'_B. Since the potentials must be the same:

$$\frac{kQ'_A}{R} = \frac{kQ'_B}{2R}$$

Simplifying:

$$Q'_A = \frac{1}{2}Q'_B$$ ½

Step 3: Total Charge After Connection The total charge before the balls were connected is:

$$Q_{\text{total}} = Q_A + Q_B = 8\pi R^2 + 48\pi R^2 = 56\pi R^2$$ ½

Since charge is conserved, after the balls are connected, the total charge is the sum of the charges on the two balls:

$$Q'_A + Q'_B = 56\pi R^2$$

Using the relation $Q'_A = \frac{1}{2}Q'_B$, we substitute into the above equation:

$$\frac{1}{2}Q'_B + Q'_B = 56\pi R^2$$ ½

$$\frac{3}{2}Q'_B = 56\pi R^2$$

$$Q'_B = \frac{2}{3} \cdot 56\pi R^2 = 37.33\pi R^2$$

Thus, the charge on ball B is $Q'_B = 37.33\pi R^2$.

Now, using $Q'_A = \frac{1}{2}Q'_B$:

$$Q'_A = \frac{1}{2} \cdot 37.33\pi R^2 = 18.67\pi R^2$$

Step 4: Final Charge Densities Finally, the charge densities on A and B are given by:

For ball A:

$$\sigma'_A = \frac{Q'_A}{4\pi R^2} = \frac{18.67\pi R^2}{4\pi R^2} = 4.67$$ ½

For ball B:

1/2

$$\sigma'_B = \frac{Q'_B}{4\pi(2R)^2} = \frac{37.33\pi R^2}{16\pi R^2} = 2.33$$

Thus, the charge densities after the balls are connected by the wire are: - $\sigma'_A = 4.67$ - $\sigma'_B = 2.33$

Quick Tip

When two conductors are connected by a wire, the final charge densities are determined by the conservation of charge and the condition that the potentials on both spheres are equal. The radius of the spheres plays a crucial role in determining the final charge densities.

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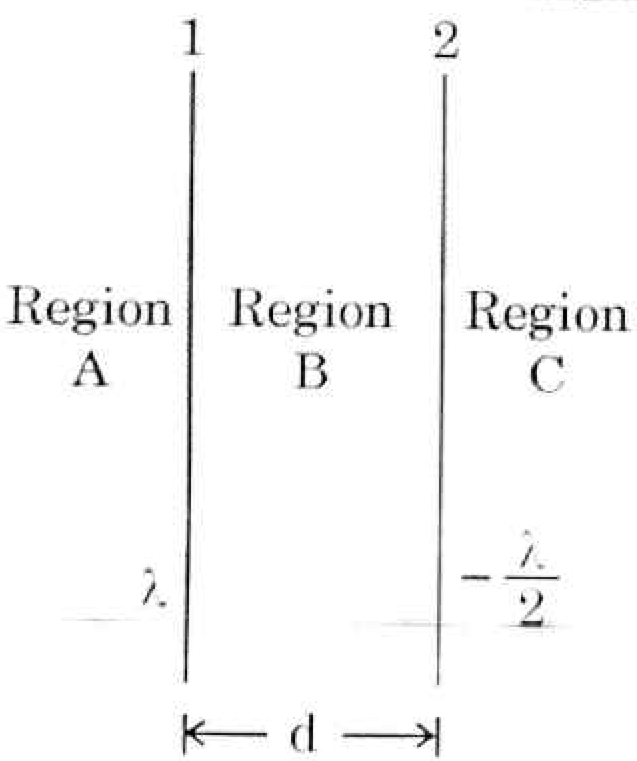
28. (b) Two infinitely long straight wires '1' and '2' are placed d distance apart, parallel to each other, as shown in the figure. They are uniformly charged having charge densities λ and $-\frac{\lambda}{2}$ respectively. Locate the position of the point from wire '1' at which the net electric field is zero and identify the region in which it lies.



Solution:

We are given two infinite straight wires with charge densities: - Wire 1: Charge density λ, - Wire 2: Charge density $-\frac{\lambda}{2}$.

We need to find the position of the point from wire 1 where the net electric field is zero.

Step 1: Electric Fields Due to Infinite Wires The electric field at a distance r from an infinitely long charged wire with charge density λ is given by:

$$E = \frac{2k_e|\lambda|}{r}$$ 1/2

Where: - k_e is Coulomb's constant, - λ is the charge density of the wire, - r is the distance from the wire.

For wire 1 (charge density λ), the electric field at a point at a distance x from wire 1 is:

$$E_1 = \frac{2k_e\lambda}{x}$$ 1/2

For wire 2 (charge density $-\frac{\lambda}{2}$), the electric field at a distance $(d-x)$ from wire 2 is:

$$E_2 = \frac{2k_e\left|-\frac{\lambda}{2}\right|}{d-x} = \frac{k_e\lambda}{d-x}$$ 1/2

Step 2: Condition for Zero Electric Field For the electric field to be zero at some point, the magnitudes of the electric fields due to the two wires must be equal and opposite in direction. Thus, we set the magnitudes of E_1 and E_2 equal:

$$\frac{2k_e\lambda}{x} = \frac{k_e\lambda}{d-x}$$ 1/2

Simplifying:

$$\frac{2}{x} = \frac{1}{d-x}$$

Cross-multiply:

$$2(d-x) = x$$

$$2d - 2x = x$$

$$2d = 3x$$

1/2

$$x = \frac{2d}{3}$$

Thus, the point where the electric field is zero is at a distance of $\frac{2d}{3}$ from wire 1.

Step 3: Identifying the Region - The distance $x = \frac{2d}{3}$ lies in Region B, which is the region between the two wires. - Therefore, the point where the electric field is zero lies in Region B. 1/2

Quick Tip

The electric field due to an infinite charged wire decreases with distance from the wire. For two wires with opposite charges, the electric fields will cancel each other out at a point where the magnitudes are equal but opposite in direction.

Section-D

29. Read the following paragraphs and answer the questions that follow.

A galvanometer is an instrument used to show the direction and strength of the current passing through it. In a galvanometer, a coil placed in a magnetic field experiences a torque and hence gets deflected when a current passes through it. The name is derived from the surname of Italian scientist L. Galvani, who in 1791 discovered that electric current makes a dead frog's leg jerk. A spring attached to the coil provides a counter torque.

In equilibrium, the deflecting torque is balanced by the restoring torque of the spring, and we have:

$$NBAI = k\phi$$

where

- N is the total number of turns in the coil
- A is the area of cross-section of each turn
- B is the radial magnetic field

- k is the torsional constant of the spring
- ϕ is the angular deflection of the coil

As the current (I_g) which produces full scale deflection in the galvanometer is very small, the galvanometer cannot as such be used to measure current in electric circuits. A small resistance, called shunt, of a suitable value is connected with the galvanometer to convert it into an ammeter of desired range. By using a higher resistance, a galvanometer can also be converted into a voltmeter.

(i) The value of the current sensitivity of a galvanometer is given by:

(A) $\frac{k}{NBA}$
(B) $\frac{NBA}{k}$
(C) $\frac{kBA}{N}$
(D) $\frac{kNB}{A}$

(1)

Correct Answer: (B) $\frac{NBA}{k}$

Solution: The current sensitivity of a galvanometer is defined as the deflection produced per unit current, and the expression for it is:

$$\text{Current Sensitivity} = \frac{NBA}{k}$$

Where: - k is a constant that depends on the galvanometer, - N is the number of turns in the coil, - B is the magnetic field, - A is the area of the coil.
Thus, the correct option is $\frac{NBA}{k}$.

Quick Tip

The current sensitivity of a galvanometer indicates how much deflection occurs per unit current. It is directly proportional to the number of turns of the coil, the magnetic field strength, and the area of the coil.

(ii) A galvanometer of resistance 6 Ω shows full scale deflection for a current of 0.2 A. The value of shunt to be used with this galvanometer to convert it into an ammeter of range (0 – 5 A) is:

(A) 0.25 Ω

(B) 0.30 Ω

(C) 0.50 Ω

(D) 6.0 Ω

Correct Answer: (A) 0.25 Ω — (1)

Solution:

To convert a galvanometer into an ammeter, a shunt resistor is connected in parallel with the galvanometer. The value of the shunt resistor is calculated using the following formula:

$$I_{\text{max}} = \frac{V_g}{R_g}$$

Where: - I_{max} is the full-scale current for the ammeter, - V_g is the voltage across the galvanometer ~~at full-scale, -~~ R_g is the resistance of the galvanometer.

For a galvanometer with resistance $R_g = 6\,\Omega$ and full-scale deflection current $I_g = 0.2\,A$, the voltage across the galvanometer is:

$$V_g = I_g \cdot R_g = 0.2 \times 6 = 1.2\,\text{V}$$

Now, to convert this galvanometer into an ammeter of range (0 – 5 A), the voltage across the galvanometer must remain the same, and the current that passes through the shunt resistor should be:

$$I_{\text{max}} = 5\,A$$

The current through the shunt resistor, I_s, will be:

$$I_s = I_{\text{max}} - I_g = 5 - 0.2 = 4.8\,A$$

The value of the shunt resistor R_s can be calculated using Ohm's law:

$$R_s = \frac{V_g}{I_s} = \frac{1.2}{4.8} = 0.25\,\Omega$$

Thus, the value of the shunt resistor is 0.25 Ω.

Quick Tip

To convert a galvanometer into an ammeter, the shunt resistor is used to bypass the excess current. The value of the shunt is determined based on the maximum current to be measured and the full-scale deflection current of the galvanometer.

(iii) The value of resistance of the ammeter in case (ii) will be:

(A) 0.20 Ω

(B) 0.24 Ω

(C) 6.0 Ω

(D) 6.25 Ω

Correct Answer: (B) 0.24 Ω — (1)

Solution: In part (ii), we calculated the value of the shunt resistor to be 0.25 Ω. Now, we need to calculate the total resistance of the ammeter, which consists of the galvanometer resistance and the shunt resistor in parallel.

The total resistance R_{total} of the ammeter is given by the parallel combination of the galvanometer resistance R_g and the shunt resistance R_s:

$$\frac{1}{R_{total}} = \frac{1}{R_g} + \frac{1}{R_s}$$

Substituting the values $R_g = 6\,\Omega$ and $R_s = 0.25\,\Omega$:

$$\frac{1}{R_{total}} = \frac{1}{6} + \frac{1}{0.25} = \frac{1}{6} + 4$$

$$\frac{1}{R_{total}} = \frac{1}{6} + \frac{24}{6} = \frac{25}{6}$$

$$R_{total} = \frac{6}{25} = 0.24\,\Omega$$

Thus, the total resistance of the ammeter is $0.24\,\Omega$.

Quick Tip

When combining resistors in parallel, the total resistance is always less than the smallest individual resistance. For an ammeter, the total resistance is determined by the parallel combination of the galvanometer resistance and the shunt resistor.

(iv)(a) A galvanometer is converted into a voltmeter of range (0 – V) by connecting with it, a resistance R_1. If R_1 is replaced by R_2, the range becomes (0 – 2 V). The resistance of the galvanometer is:

(A) $R_2 - 2R_1$

(B) $R_2 - R_1$

(C) $R_1 + R_2$

(D) $R_1 - 2R_2$

Correct Answer: (C) $R_1 + R_2$ — (1)

Solution:

The resistance of a galvanometer is R_g, and to convert it into a voltmeter, we add a series resistance. The range of the voltmeter is given by:

$$V = I_g \cdot (R_g + R)$$

Where: - V is the range of the voltmeter, - I_g is the current at full scale deflection for the galvanometer, - R_g is the resistance of the galvanometer, - R is the series resistance added. For range $0 - V$, the series resistance is R_1, so the total resistance is $R_g + R_1$. For range $0 - 2V$, the series resistance is R_2, so the total resistance is $R_g + R_2$.

Since the voltage is doubled when R_1 is replaced by R_2, we have:

$$\frac{R_g + R_2}{R_g + R_1} = 2$$

Solving for R_g:

$$R_g + R_2 = 2(R_g + R_1)$$

$$R_g + R_2 = 2R_g + 2R_1$$

$$R_2 = R_g + 2R_1$$

Thus, the resistance of the galvanometer is:

$$R_g = R_2 - 2R_1$$

Therefore, the correct answer is $R_2 - 2R_1$.

Quick Tip

When converting a galvanometer into a voltmeter, the series resistance affects the range. The relationship between the resistance and the range is linear, and the resistance of the galvanometer can be derived from the change in the series resistance.

OR,

(b) A current of 5 mA flows through a galvanometer. Its coil has 100 turns, each of area of cross-section 18 cm^2 and is suspended in a magnetic field of 0.20 T. The deflecting torque acting on the coil will be:

(A) 3.6×10^{-3} Nm

(B) 1.8×10^{-4} Nm

(C) 2.4×10^{-3} Nm

(D) 1.2×10^{-4} Nm

Correct Answer: (B) 1.8×10^{-4} Nm

–(

Solution:

The deflecting torque T on a coil in a magnetic field is given by the formula:

$$T = nBAI$$

Where: - n is the number of turns, - B is the magnetic field strength, - A is the area of the coil, - I is the current flowing through the coil.

Substituting the given values: - $n = 100$, - $B = 0.20\,\text{T}$, - $A = 18\,\text{cm}^2 = 18 \times 10^{-4}\,\text{m}^2$, - $I = 5\,\text{mA} = 5 \times 10^{-3}\,\text{A}$,

The torque is:

$$T = 100 \times 0.20 \times 18 \times 10^{-4} \times 5 \times 10^{-3}$$

$$T = 1.8 \times 10^{-4}\,\text{Nm}$$

Thus, the deflecting torque acting on the coil is $1.8 \times 10^{-4}\,\text{Nm}$.

Quick Tip

The deflecting torque on a coil in a magnetic field depends on the number of turns, the magnetic field strength, the area of the coil, and the current flowing through it. The formula $T = nBAI$ is used to calculate the torque.

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30. Read the following paragraphs and answer the questions that follow.

Einstein explained the photoelectric effect on the basis of Planck's quantum theory, where light travels in the form of small bundles of energy called photons. The energy of each photon is $h\nu$, where ν is the frequency of incident light and h is Planck's constant. The number of photons in a beam of light determines the intensity of the incident light. A photon incident on a metal surface transfers its total energy $h\nu$ to a free electron in the metal. A part of this energy is used in ejecting the electron from the metal and is called its work function. The rest of the energy is carried by the ejected electron as its kinetic energy.

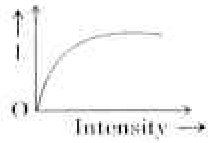
(i) Which of the following graphs shows the variation of photoelectric current I with the intensity of light?

(A)

(B)

(C)

(D)



Correct Answer: (A) Linear increase of current with intensity — (1)

Solution:

In the photoelectric effect, when light of frequency higher than the threshold frequency is incident on the metal surface, photoelectrons are ejected. The number of photoelectrons emitted is directly proportional to the intensity of light because intensity represents the number of photons striking the surface per unit time.

The photoelectric current I is proportional to the number of photoelectrons emitted, which in turn is directly proportional to the intensity of light.

Thus, the graph of photoelectric current I with intensity of light is a straight line, as shown in Option (A).

Quick Tip

In the photoelectric effect, the intensity of light determines the number of photons and thus the number of ejected electrons, leading to a linear increase in the photoelectric current with the intensity.

(ii) When the frequency of the incident light is increased without changing its intensity, the saturation current:

(A) increases linearly

(B) decreases

(C) increases non-linearly

(D) remains the same

Correct Answer: (D) remains the same — (1)

Solution:

In the photoelectric effect, the saturation current depends on the number of photoelectrons emitted, which is determined by the intensity of the light. The saturation current is independent of the frequency of the incident light, as long as the frequency is above the threshold frequency required to eject electrons.

When the frequency of the light is increased (above the threshold frequency) without changing its intensity, the energy of each photon increases, but the number of photons (and hence the number of ejected electrons) remains the same because the intensity (which is related to the number of photons) is unchanged. As a result, the saturation current remains the same.

Thus, the correct answer is (D): The saturation current remains the same.

Quick Tip

The saturation current in the photoelectric effect depends on the intensity of light, not its frequency. Increasing the frequency (above the threshold frequency) does not change the saturation current, but increases the energy of each emitted photon.

(iii) Which of the following graphs can be used to obtain the value of Planck's constant?

(A) **Photocurrent versus Intensity of incident light**

(B) **Photocurrent versus Frequency of incident light**

(C) **Cut-off potential versus Frequency of incident light**

(D) **Cut-off potential versus Intensity of incident light**

Correct Answer: **(C) Cut-off potential versus Frequency of incident light** — (1)

Solution:

According to Einstein's photoelectric equation:

$$E_k = hf - \phi$$

Where: - E_k is the kinetic energy of the emitted electrons, - h is Planck's constant, - f is the frequency of the incident light, - ϕ is the work function of the material.

The cut-off potential V_0 is related to the maximum kinetic energy by:

$$E_k = eV_0$$

Thus, the cut-off potential is proportional to the frequency of the incident light. The graph of V_0 (cut-off potential) versus f (frequency) will be a straight line, and the slope of this line will give the value of Planck's constant h.

Quick Tip

To determine Planck's constant experimentally, a graph of cut-off potential versus frequency of incident light can be plotted. The slope of this graph gives Planck's constant.

(iv)(a) Red light, yellow light, and blue light of the same intensity are incident on a metal surface successively. K_R**,** K_Y**, and** K_B **represent the maximum kinetic energy of photoelectrons respectively, then:**

(A) $K_R > K_Y > K_B$

(B) $K_Y > K_B > K_R$

(C) $K_B > K_Y > K_R$

(D) $K_R > K_B > K_Y$

—(1)

Correct Answer: (C) $K_B > K_Y > K_R$

Solution:

According to Einstein's photoelectric equation:

$$K_{\max} = h\nu - \phi$$

where $K_{\max}$ is the maximum kinetic energy of the emitted photoelectrons, h is Planck's constant, ν is the frequency of the incident light, and φ is the work function of the metal. Since the intensity of the light is the same for all three colors, the only factor influencing the kinetic energy is the frequency of the incident light. The energy of a photon is given by $E = h\nu$, so:

- Red light has the lowest frequency, hence the lowest energy per photon, and thus the lowest kinetic energy of the emitted electrons. - Yellow light has a higher frequency and energy per photon compared to red, so the kinetic energy will be higher than for red. - Blue light has the highest frequency and energy per photon, so the kinetic energy of the emitted photoelectrons will be the highest.

Thus, the kinetic energies follow the order:

$$K_B > K_Y > K_R$$

Therefore, the correct answer is (C): $K_B > K_Y > K_R$.

Quick Tip

The kinetic energy of photoelectrons increases with the frequency of the incident light, provided the light has energy above the work function of the material. Higher frequency light (blue) leads to higher kinetic energy than lower frequency light (red).

(b) Which of the following metals exhibits photoelectric effect with visible light?

(A) **Caesium**

(B) **Zinc**

(C) **Cadmium**

(D) **Magnesium**

Correct Answer: (A) **Caesium** — (1)

Solution:

Among the given metals, **Caesium** (option A) exhibits the photoelectric effect with visible light. The work function of cesium is low enough that visible light has sufficient energy to dislodge electrons from its surface.

For the other metals listed (Zinc, Cadmium, and Magnesium), the work function is higher than that of visible light, so they do not exhibit the photoelectric effect with visible light.

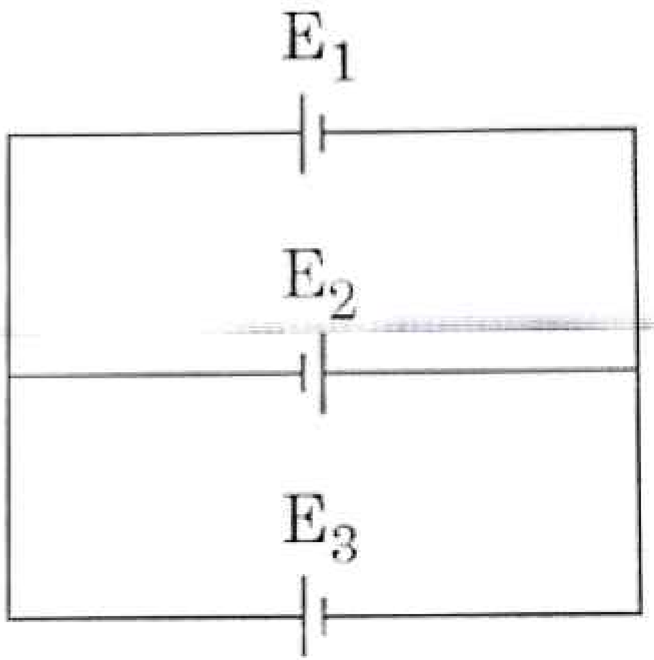
Therefore, the correct answer is **Caesium**.

Quick Tip

Caesium has the lowest work function of the metals listed, which allows it to exhibit the photoelectric effect with visible light. The photoelectric effect is observed when the energy of the incident photons is greater than or equal to the work function of the material.

Section-E

31. (a)(i) Three batteries E1, E2, and E3 of emfs and internal resistances (4 V, 2 Ω), (2 V, 4 Ω) and (6 V, 2 Ω) respectively are connected as shown in the figure. Find the values of the currents passing through batteries E1, E2, and E3.



Solution:

Let's denote the current flowing through the batteries as I_1, I_2, and I_3. We will use Kirchhoff's Laws to solve for the currents in the circuit.

Step 1: Assign direction to the currents.

Assume the directions of the currents as shown in the figure.

Step 2: Apply Kirchhoff's Voltage Law (KVL).

For battery E1, the loop equation is:

$$E_1 - I_1R_1 - I_2R_2 = 0$$ — 1/2

For battery E2:

$$E_2 - I_2R_2 - I_1R_1 = 0$$ $-\frac{1}{2}$

For battery E3:

$$E_3 - I_3R_3 = 0$$ $-\frac{1}{2}$

Substitute the values of $E_1 = 4\,V$, $E_2 = 2\,V$, $E_3 = 6\,V$, and the resistances $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 2\,\Omega$ into the equations.

Step 3: Solve the system of equations.

We now have a system of linear equations. Solving them will give the values of I_1, I_2, and I_3.

The values of the currents passing through the batteries are:

I_1 = Value of current through battery E1 (A) $-\frac{1}{2}$

I_2 = Value of current through battery E2 (A) $-\frac{1}{2}$

I_3 = Value of current through battery E3 (A) $-\frac{1}{2}$

Quick Tip

Kirchhoff's Voltage Law (KVL) states that the sum of all voltage drops (or rises) in a closed loop must be zero. Use this law to solve for the unknown currents in the circuit.

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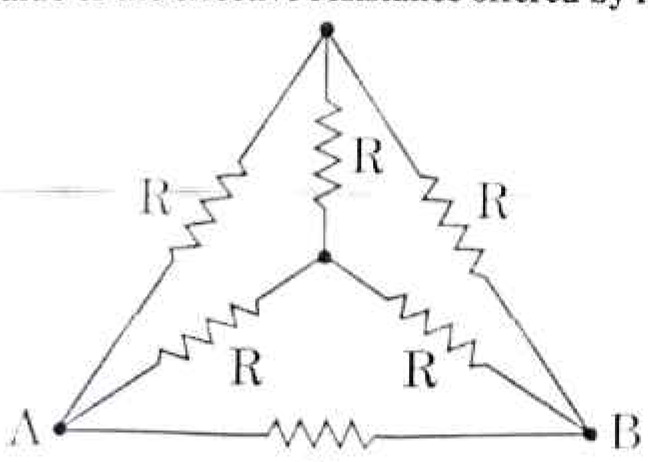
31. (ii) The ends of six wires, each of resistance R (= 10 Ω) are joined as shown in the figure. The points A and B of the arrangement are connected in a circuit. Find the value of the effective resistance offered by it to the circuit.



Solution:

The given circuit is a symmetric triangle, where each side has a resistance of R. We need to find the effective resistance between points A and B.

Step 1: Resistors in Series

Each set of two resistors in series will give:

$$R_{eq1} = R + R = 2R$$ — ½

Step 2: Resistors in Parallel

The three $2R$ resistors are now in parallel. Using the formula for resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R} = \frac{3}{2R}$$ — ½

Thus, the effective resistance is:

$$R_{eq} = \frac{2R}{3}$$ — ½

Step 3: Substituting the value of R

Given that $R = 10\,\Omega$, we have:

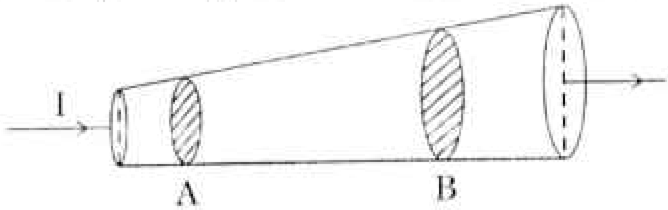
$$R_{eq} = \frac{2 \times 10}{3} = 6.67\,\Omega$$ — ½

Quick Tip

In symmetric circuits, identify series and parallel combinations systematically to simplify the network step by step.

---

31. (b) (i) Current I (= 1 A) is passing through a copper rod (n = 8.5×10^{28} m^{-3}) of varying cross-sections as shown in the figure. The areas of cross-section at points A and B along its length are 1.0×10^{-7} m^2 and 2.0×10^{-7} m^2 respectively. Calculate:



(I) the ratio of electric fields at points A and B.

(II) the drift velocity of free electrons at point B.

Solution:

Let the current $I = 1$ A pass through the copper rod. The electric field E and drift velocity v_d are related to the current by:

$$I = nAev_d$$

where: - n is the number of free electrons per unit volume, - A is the cross-sectional area, - e is the charge of an electron, - v_d is the drift velocity.

Also, the electric field E is related to the drift velocity by:

$$E = \rho J$$

1/2

where ρ is the resistivity of the material and J is the current density.

Now, the drift velocity and electric field are inversely proportional to the area of cross-section, meaning the electric field at point A and point B can be compared as:

$$\frac{E_A}{E_B} = \frac{A_B}{A_A}$$

— 1/2

Substituting the areas:

$$\frac{E_A}{E_B} = \frac{2.0 \times 10^{-7}}{1.0 \times 10^{-7}} = 2$$

Thus, the ratio of electric fields at points A and B is:

$$\frac{E_A}{E_B} = 2$$

— 1/2

Now, to calculate the drift velocity at point B, we can use the equation:

$$v_{dB} = \frac{I}{nA_Be}$$

1/2

Substituting the given values:

$$v_{dB} = \frac{1}{8.5 \times 10^{28} \times 2.0 \times 10^{-7} \times 1.6 \times 10^{-19}} \approx 3.7 \times 10^{-4}\,\text{m/s}$$

1/2

Thus, the drift velocity at point B is:

$$v_{dB} \approx 3.7 \times 10^{-4}\,\text{m/s}$$

Quick Tip

In problems involving current through varying cross-sections, use the relationship between current, drift velocity, and electric field to solve for quantities of interest. Remember that the electric field is inversely proportional to the cross-sectional area.

---

31. (ii) Two point charges $q_1 = 16\,\mu C$ and $q_2 = 1\,\mu C$ are placed at points $\vec{r}_1 = (3\,\mathbf{m})\hat{i}$ and $\vec{r}_2 = (4\,\mathbf{m})\hat{j}$. Find the net electric field $\vec{E}$ at point $\vec{r} = (3\,\mathbf{m})\hat{i} + (4\,\mathbf{m})\hat{j}$.

Solution:

We are given two point charges and we need to find the net electric field at a point $\vec{r} = 3\hat{i} + 4\hat{j}$ due to the two charges.

The electric field due to a point charge is given by Coulomb's law:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\hat{r}$$ 1/2

where: - q is the charge, - r is the distance from the charge to the point of interest, - $\hat{r}$ is the unit vector pointing from the charge to the point, - ε_0 is the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$.

Electric field due to q_1 at point $\vec{r}$:

First, calculate the distance r_1 between q_1 and point $\vec{r}$. Since q_1 is at $(3, 0)$ and point $\vec{r}$ is at $(3, 4)$:

$$r_1 = \sqrt{(3-3)^2 + (4-0)^2} = \sqrt{16} = 4\,\text{m}$$ 1/4

Next, find the unit vector $\hat{r_1}$:

$$\hat{r_1} = \frac{\vec{r} - \vec{r_1}}{r_1} = \frac{(3\hat{i} + 4\hat{j}) - (3\hat{i})}{4} = \hat{j}$$ 1/4

Now, using Coulomb's law for q_1:

$$\vec{E_1} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1^2}\hat{r_1} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{16 \times 10^{-6}}{(4)^2}\hat{j} = \frac{16 \times 10^{-6}}{16\pi\varepsilon_0}\hat{j}$$ 1/4

Electric field due to q_2 at point $\vec{r}$:

Similarly, calculate the distance r_2 between q_2 and point $\vec{r}$:

$$r_2 = \sqrt{(3-0)^2 + (4-4)^2} = \sqrt{9} = 3\,\text{m}$$ 1/4

Find the unit vector $\hat{r_2}$:

$$\hat{r_2} = \frac{\vec{r} - \vec{r_2}}{r_2} = \frac{(3\hat{i} + 4\hat{j}) - (0\hat{i} + 4\hat{j})}{3} = \frac{3\hat{i}}{3} = \hat{i}$$ 1/4

Now, using Coulomb's law for q_2:

$$\vec{E_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2^2}\hat{r_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1 \times 10^{-6}}{(3)^2}\hat{i} = \frac{1 \times 10^{-6}}{9\pi\varepsilon_0}\hat{i}$$ 1/4

Net electric field at point $\vec{r}$:

The total electric field $\vec{E}$ at point $\vec{r}$ is the vector sum of $\vec{E_1}$ and $\vec{E_2}$:

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{16 \times 10^{-6}}{16\pi\varepsilon_0}\hat{j} + \frac{1 \times 10^{-6}}{9\pi\varepsilon_0}\hat{i}$$ 1/2

Thus, the net electric field at point $\vec{r}$ is:

$$\vec{E} = \frac{1 \times 10^{-6}}{9\pi\varepsilon_0}\hat{i} + \frac{16 \times 10^{-6}}{16\pi\varepsilon_0}\hat{j}$$

Quick Tip

When calculating the net electric field from multiple charges, calculate the electric field due to each charge separately and then combine the results vectorially, taking care of the direction.

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32. (i) Define self-inductance of a coil. Derive the expression for the energy required to build up a current I in a coil of self-inductance L.

Solution:

Self-inductance of a coil is a property of the coil that describes its ability to oppose changes in current flowing through it. When a current I flows through a coil, it creates a magnetic flux. The self-inductance L is defined as the ratio of the induced emf (electromotive force) in the coil to the rate of change of current. Mathematically, it is given by: (1)

$$L = \frac{N\Phi}{I}$$

where: - N is the number of turns in the coil, - Φ is the magnetic flux through each turn, - I is the current flowing through the coil.

Now, let's derive the expression for the energy required to build up a current I in a coil of self-inductance L.

Energy required to build up a current:

The work dW required to increase the current I by an infinitesimal amount dI in the coil is given by the product of the induced emf $\mathcal{E}$ and the infinitesimal current change dI:

$$dW = \mathcal{E} \cdot dI$$ — 1/2

From Faraday's law, the induced emf is:

$$\mathcal{E} = -L\frac{dI}{dt}$$ — 1/2

Substituting this in the above equation:

$$dW = -L\frac{dI}{dt} \cdot dI$$

Since dI/dt is the rate of change of current, we need to integrate to find the total work required to build up the current from 0 to I.

The total work done (or energy) to establish the current is:

$$W = \int_0^I L\,I\,dI$$ — 1/2

Solving the integral:

$$W = \frac{1}{2}LI^2$$ —1/2

Thus, the energy required to build up a current I in a coil of self-inductance L is:

$$\boxed{W = \frac{1}{2}LI^2}$$

This is the expression for the energy stored in the magnetic field of the coil as the current increases.

Quick Tip

The energy required to build up a current in a coil is proportional to the square of the current and the self-inductance. This energy is stored as magnetic potential energy in the coil.

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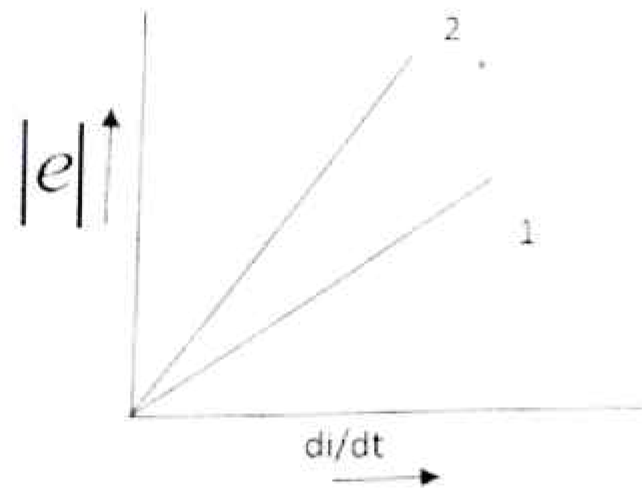
32. (ii) The currents passing through two inductors of self-inductances 10 mH and 20 mH increase with time at the same rate. Draw graphs showing the variation of:

(I) The magnitude of emf induced with the rate of change of current in each inductor:

Since the induced emf $\mathcal{E}$ in an inductor is related to the rate of change of current $\frac{dI}{dt}$ by the formula:

$$\mathcal{E} = L\frac{dI}{dt}$$

where L is the inductance of the inductor. Since the currents in both inductors increase with the same rate, the emf is proportional to the inductance. Therefore, for an inductor with a larger inductance, the induced emf will be higher for the same rate of change of current. Thus, the graph will show the emf being proportional to L, with the emf being twice as large for the inductor with self-inductance of 20 mH compared to 10 mH.



(1)

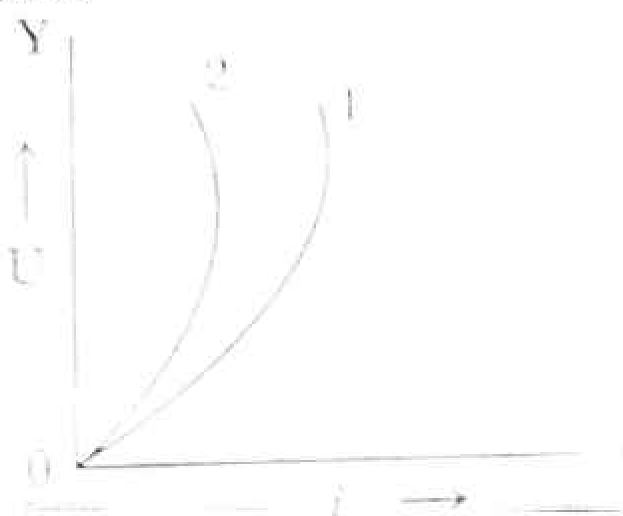
(II) The energy stored in each inductor with the current flowing through it:

The energy stored in an inductor is given by:

$$W = \frac{1}{2}LI^2$$

where W is the energy, L is the inductance, and I is the current. Since the inductance of the second inductor is twice that of the first, for the same current, the energy stored in the second inductor will be twice the energy stored in the first one.

Thus, the graph will show a quadratic relationship between current and energy stored. The curve for the inductor with larger inductance will be higher by a factor of 2 for the same current.



(1)

Quick Tip

The induced emf in an inductor is directly proportional to the rate of change of current ($\frac{dI}{dt}$) and the inductance (L). The energy stored in an inductor is proportional to the square of the current (I^2) and the inductance (L).

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32. (b) (i) Define the term mutual inductance. Deduce the expression for the mutual inductance of two long coaxial solenoids of the same length having different radii and different number of turns.

Solution:

Mutual inductance M between two coils is the property of the system that quantifies how much the magnetic flux produced by one coil links with the second coil. It is defined as the ratio of the induced emf in coil 2 to the rate of change of current in coil 1. (1)

The mutual inductance between two coaxial solenoids can be derived as follows:

Let: - N_1 be the number of turns of the first solenoid, - N_2 be the number of turns of the second solenoid, - L be the length of each solenoid, - R_1 be the radius of the first solenoid, - R_2 be the radius of the second solenoid, - μ be the permeability of the material inside the solenoids.

The magnetic field produced by the first solenoid inside it is given by Ampere's Law:

$$B_1 = \frac{\mu N_1 I_1}{L}$$

The flux linkage of the second solenoid is:

$$\Phi_2 = B_1 \cdot A_2$$

where A_2 is the cross-sectional area of the second solenoid, given by $A_2 = \pi R_2^2$.

Thus, the flux linkage becomes:

$$\Phi_2 = \frac{\mu N_1 I_1 \pi R_2^2}{L}$$ $-1/2$

The induced emf in the second solenoid is:

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_2}{dt}$$ $-1/2$

The mutual inductance M is then defined as:

$$M = \frac{\mathcal{E}_2}{dI_1/dt}$$ $1/2$

Substituting for $\mathcal{E}_2$:

$$M = \frac{\mu N_1 N_2 \pi R_2^2}{L}$$ $-1/2$

This is the expression for the mutual inductance between two coaxial solenoids.

Quick Tip

Mutual inductance quantifies the relationship between two coils and is directly proportional to the number of turns in each coil and the cross-sectional area of the second coil.

---

32. (b) (ii) The current through an inductor is uniformly increased from zero to 2 A in 40 s. An emf of 5 mV is induced during this period. Find the flux linked with the inductor at t = 10 s.

Solution:

We know the emf induced in an inductor is given by:

$$\mathcal{E} = L\frac{dI}{dt}$$ $1/2$

where L is the inductance of the inductor, I is the current, and $\frac{dI}{dt}$ is the rate of change of current.

The current increases uniformly from 0 to 2 A in 40 seconds, so the rate of change of current is:

$$\frac{dI}{dt} = \frac{2\text{ A}}{40\text{ s}} = 0.05\text{ A/s}$$

Substituting this into the equation for emf:

$$\mathcal{E} = L \times 0.05$$

Given that the induced emf is 5 mV or 5×10^{-3} V, we can solve for L:

$$5 \times 10^{-3} = L \times 0.05$$

$$L = \frac{5 \times 10^{-3}}{0.05} = 10^{-1}\,\mathrm{H} = 0.1\,\mathrm{H}$$ 1/2

Now, the flux Φ linked with the inductor at any time t is given by:

$$\Phi = L \times I$$

At $t = 10$ s, the current is:

$$I = \frac{2\,\mathrm{A}}{40\,\mathrm{s}} \times 10\,\mathrm{s} = 0.5\,\mathrm{A}$$ 1/2

Thus, the flux at $t = 10\,\mathrm{s}$ is:

$$\Phi = 0.1 \times 0.5 = 0.05\,\mathrm{Wb}$$ 1/2

Quick Tip

The flux linked with an inductor is the product of the inductance and the current flowing through it. The induced emf is proportional to the rate of change of current.

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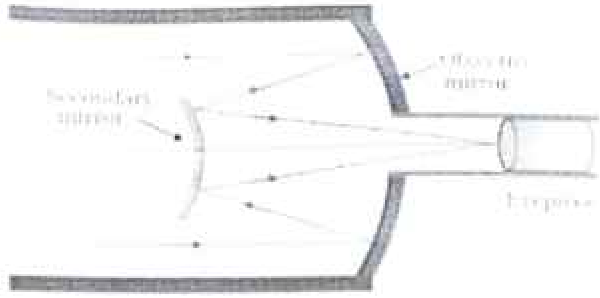
33. (a) (i) Draw a ray diagram of a reflecting telescope (Cassegrain) and explain the formation of the image. State two important advantages that a reflecting telescope has over a refracting telescope.

Solution:

A reflecting telescope uses mirrors to gather and focus light. The Cassegrain type has two mirrors: a primary concave mirror and a secondary convex mirror. The primary mirror collects light from distant objects, and the secondary mirror reflects the light to an eyepiece located at the focal point of the primary mirror.



(1)

In the diagram: - The rays from a distant object are reflected by the concave primary mirror. - The reflected rays converge at the focus of the primary mirror, and the secondary convex mirror reflects these rays towards the eyepiece. - The eyepiece focuses the rays into a clear image.

1. **No chromatic aberration:** Reflecting telescopes use mirrors instead of lenses, so they don't suffer from chromatic aberration, which causes color distortion in refracting telescopes.
2. **Larger apertures:** Reflecting telescopes can have much larger apertures than refracting telescopes, allowing them to collect more light and observe fainter objects.

Quick Tip

In reflecting telescopes, mirrors are used instead of lenses. This helps avoid chromatic aberration and allows for larger apertures.

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33. (a) (ii)

In a refracting telescope, the focal length of the objective is 50 times the focal length of the eyepiece. When the final image is formed at infinity, the length of the tube is 102 cm. Find the focal lengths of the two lenses.

Solution:

We know that the total length of the telescope tube is the sum of the focal lengths of the objective lens and the eyepiece lens. Let: - f_o be the focal length of the objective lens, - f_e be the focal length of the eyepiece lens.

We are given:

$$f_o = 50f_e$$ 1/2

and

$$f_o + f_e = 102\,\text{cm}$$ 1/2

Substituting $f_o = 50f_e$ into the second equation:

$$50f_e + f_e = 102$$

$$51f_e = 102$$

$$f_e = \frac{102}{51} = 2\,\text{cm}$$ 1/2

Now, using $f_o = 50f_e$:

$$f_o = 50 \times 2 = 100\,\text{cm}$$ 1/2

Thus, the focal lengths of the two lenses are: - $f_o = 100\,\text{cm}$ (objective lens), - $f_e = 2\,\text{cm}$ (eyepiece lens).

Quick Tip

In a refracting telescope, the total tube length is the sum of the focal lengths of the objective and eyepiece lenses. If the objective's focal length is 50 times the eyepiece's focal length, you can use this relationship to find the focal lengths.

OR,

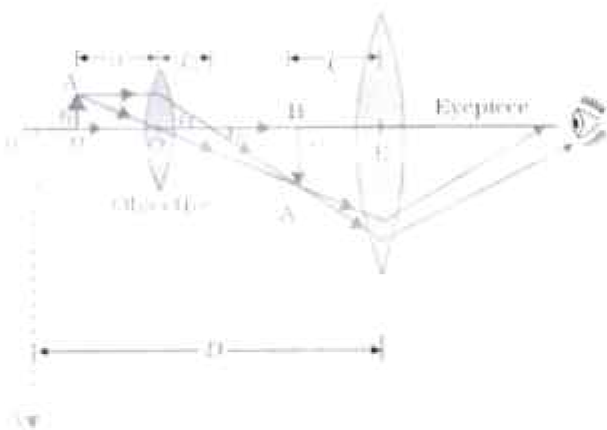
33. (b) (i) Write any two advantages of a compound microscope over a simple microscope. Draw a ray diagram for the image formation at the near point by a compound microscope and explain it.

Solution:

A compound microscope consists of two lenses: the objective lens and the eyepiece lens. The objective lens forms a real, inverted image of the object at a short distance, which is further magnified by the eyepiece to form the final image. The advantages of a compound microscope over a simple microscope are:

1. **Higher magnification:** A compound microscope has two lenses working together (objective and eyepiece), providing much higher magnification than a simple microscope (which typically uses only one lens). 1/2

2. **Improved clarity and resolution:** Due to the multiple lenses, a compound microscope offers better resolution and can focus on much smaller details compared to a simple microscope. 1/2



— (1)

In the diagram: - The object is placed at a distance less than the focal length of the objective lens, causing it to form an intermediate real image. - This intermediate image acts as an object for the eyepiece lens, which further magnifies the image, and it is viewed at the near point. (1)

Quick Tip

A compound microscope offers higher magnification and better resolution by using two lenses to form and further magnify the image.

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33. (b) (ii) A thin plano-concave lens with its curved face of radius of curvature R is made of glass of refractive index n_1. It is placed coaxially in contact with a thin equiconvex lens of same radius of curvature of refractive index n_2. Obtain the power of the combination lens.

Solution:

For a combination of lenses in contact, the effective power P is the sum of the powers of the individual lenses:

$$P_{\text{effective}} = P_1 + P_2$$

$-1/2$

where P_1 is the power of the plano-concave lens and P_2 is the power of the equiconvex lens. The power P of a lens is given by:

$$P = \frac{1}{f}$$

where f is the focal length of the lens. For a plano-concave lens with radius of curvature R and refractive index n_1:

$$\frac{1}{f_1} = \frac{n_1 - 1}{R}$$

$1/4$

For the equiconvex lens with radius of curvature R and refractive index n_2:

$$\frac{1}{f_2} = \frac{n_2 - 1}{R}$$

$1/4$

Now, the total power of the combination is:

$$P_{\text{effective}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{n_1 - 1}{R} + \frac{n_2 - 1}{R}$$

$-1/2$

$$P_{\text{effective}} = \frac{(n_1 - 1) + (n_2 - 1)}{R}$$

Thus, the effective power of the combination is:

$$P_{\text{effective}} = \frac{(n_1 + n_2 - 2)}{R}$$

$-1/2$

Quick Tip

The total power of a combination of lenses in contact is the sum of the individual powers, and for curved surfaces, the focal length depends on the radius of curvature and the refractive index.